On preferred observation basis

Sergei Viznyuk

Abstract

I show the unitarily equivalent solutions of Schrödinger equation predict different measurement results in different observation bases. I identify the criterion for choosing an observation basis in which the predicted measurement results are consistent with observed macroscopic behavior

The decoherence [1, 2, 3] has been discussed as a factor in selection of observation basis [4, 5], in which the results of a measurement on quantum object are represented, albeit the exact criteria for such selection remain elusive [6]. I show a parameter, derived from the energy spectrum of the system in a given observation basis, is a key factor in observable macroscopic change rates. The observable change rates are linked by this parameter to the preferred observation basis, thereby selecting it from otherwise unlimited set.

Consider a solution of Schrödinger equation in $\boldsymbol{H}$-matrix eigenbasis, with initial condition $\psi(0)$:

$$\psi(t) = exp\left(-i\frac{\boldsymbol{H}}{\hbar}t\right)\cdot\psi(0) = \sum_k |f_k\rangle\langle f_k|\psi(0)\rangle\cdot exp\left(-i\frac{E_k}{\hbar}t\right) \tag{1}$$

, where f_k, E_k are eigenstates and eigenvalues of $\boldsymbol{H}$-matrix, with no artificial boundaries between quantum object, measuring device, or environment. The matrix $\boldsymbol{H}$ is diagonal in its eigenbasis:

$$\boldsymbol{H} = \begin{pmatrix} E_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & E_k \end{pmatrix} \tag{2}$$

In a different observation basis $\boldsymbol{H}$-matrix transforms to

$$\boldsymbol{H}' = \langle\boldsymbol{U}|\boldsymbol{H}|\boldsymbol{U}^\dagger\rangle + i\hbar\left(\frac{\partial\boldsymbol{U}}{\partial t}\right)\boldsymbol{U}^\dagger \tag{3}$$

If I choose transformation $\boldsymbol{U}$ as

$$\boldsymbol{U} = \begin{pmatrix} exp\left(i\frac{\varepsilon_1}{\hbar}t\right) & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & exp\left(i\frac{\varepsilon_k}{\hbar}t\right) \end{pmatrix} \tag{4}$$

, the transformed $\boldsymbol{H}'$ matrix, from (3), becomes

$$\boldsymbol{H}' = \begin{pmatrix} E_1-\varepsilon_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & E_k-\varepsilon_k \end{pmatrix} = \begin{pmatrix} E_1' & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & E_k' \end{pmatrix} \tag{5}$$

The transformation (4) changed eigenvalues (5), but preserved eigenvectors, up to a phase factor, and eigenstate probabilities $P_k = |\langle f_k|\psi(0)\rangle|^2 = |\langle f_k'|\psi'(0)\rangle|^2 = P_k'$. The measurement of any observable which commutes with $\boldsymbol{H}$-matrix ought to have the same results in old and in new basis, even though the wave function (1) is different in new basis, due to the different eigenvalues (5). The difference between observation results in old and in new basis would come from conditional measurement, i.e. Born rule:

$$P(t) = |\langle\psi(t)|\psi(0)\rangle|^2 = \sum_{k,j} P_k P_j \cos\left(\frac{E_k - E_j}{\hbar} t\right) = \left\langle \boldsymbol{P} \middle| \cos\left(\frac{\boldsymbol{E}}{\hbar} t\right) \middle| \boldsymbol{P} \right\rangle$$
$$\neq P'(t) = \left\langle \boldsymbol{P} \middle| \cos\left(\frac{\boldsymbol{E}'}{\hbar} t\right) \middle| \boldsymbol{P} \right\rangle \quad (6)$$

, where $\boldsymbol{P} = P_{\{k\}}$ is the eigenstate probabilities vector; $\boldsymbol{E} = E_{kj} = E_k - E_j$; $\boldsymbol{E}' = E'_{kj} = E'_k - E'_j$. I have arrived to a situation when unitarily equivalent solutions of Schrödinger equation predict different measurement results (6). In a classical world, this would come to no surprise, as we know the state of an object looks different with respect to different reference frames. In particular, an object can be accelerating (i.e. changing its state) with respect to one reference frame, while being static in another. It is not the case with quantum object in a pure state (1), as the rate of change of pure state in any observation basis is zero [7]:

$$\left[\frac{\partial P(t)}{\partial t}\right]_{t=0} = \left[\frac{\partial P'(t)}{\partial t}\right]_{t=0} = 0 \quad (7)$$

The zero-change rate for a pure state of quantum object is expected, since the quantum object, prepared in a definite quantum state, in the absence of decoherence, will remain in pure state indefinitely. The different Born probabilities $P(t)$ vs. $P'(t)$ in (6) come not from a change in the state of the object but from a change in observation basis, linked to a change in macroscopic parameter t. In the absence of a change in the state of the object, there is no obvious choice of one observation basis over another.

It thus deems impossible to pick one observation basis over another, unless the change rate (7) is non-zero, and is consistent with classical observation. The non-zero change rate is the result of decoherence between constituent eigenstates. As has been reported elsewhere [8], the rate of change in object state is linked to $\boldsymbol{E}$-matrix (above). The antisymmetric $\boldsymbol{E}$-matrix has two non-zero purely imaginary eigenvalues which only differ in sign:

$$\mathcal{E}_{\boldsymbol{E}} = |eigenvalue(\boldsymbol{E})| \quad ; \quad \mathcal{E}_{\boldsymbol{E}}^2 = trace(\boldsymbol{E} \cdot \boldsymbol{E}^\dagger)/2 = -trace(\boldsymbol{E}^2)/2 \quad (8)$$

The rate of change in object state, correlative with a change of abstract dimensionless macroscopic parameter t, is [8]:

$$\lambda = -\left[\frac{\partial P(t)}{\partial t}\right]_{t=0} = \frac{\mathcal{E}_{\boldsymbol{E}}^2}{h} \ (nats) \ ; \ \ h = 1\ (nats) \quad (9)$$

, where $\mathcal{E}_{\boldsymbol{E}}, h$ are in $(nats)$. The expression for the change rate (9) takes a dimensional form if macroscopic parameter t is dimensional. E.g., if t is time (s), the change rate is the *transition rate*:

$$\lambda = \frac{\mathcal{E}_{\boldsymbol{E}}^2 \tau}{h^2} \ (nats/s) \quad (10)$$

, where τ $(nats \cdot s)$ is a characteristic decoherence time; $\mathcal{E}_{\boldsymbol{E}}$ is in joules (J), and h is in $(J \cdot s)$. The expression (10) for transition rate, containing $\mathcal{E}_{\boldsymbol{E}}^2$ in one form or another, is present in a number of QM artifacts, from Fermi's golden rule [9], to Planck's radiation law [10], to the predicted gravity-induced decay [11].

The $\mathcal{E}_{\boldsymbol{E}}^2$ factor must be ubiquitous to any observable macroscopic change rate. The observation basis, in which the value of $\mathcal{E}_{\boldsymbol{E}}^2$ factor, derived from $\boldsymbol{E}$-matrix, is consistent with observed macroscopic change rate, is the *preferred basis*.

The follow up question maybe asked, if it is the choice of observation basis which determines the change rate, or is it the change rate which determines the observation basis. There might not be an answer for this chicken-or-the-egg-type question, unless we assume, they both are determined by the measurement setup [4]. In which case, we also have to conclude a solution of Schrödinger equation (i.e. wave function) does not describe any physical reality, unless it is expressed in a particular basis corresponding to the measurement setup, with subsequent conclusion, that the physical reality is determined by the measurement setup, outside of which it remains a featureless unknown.

References

[1] M. Schlosshauer, "Quantum Decoherence," arXiv:1911.06282 [quant-ph], 2019. [Online]. Available: https://arxiv.org/abs/1911.06282.

[2] K. Hornberger, "Introduction to Decoherence Theory," *Lect. Notes Phys.*, pp. 221-276, 2009.

[3] W. Zurek, "Decoherence and the Transition from Quantum to Classical - Revisited," arXiv:quant-ph/0306072, 2003. [Online]. Available: https://arxiv.org/abs/quant-ph/0306072.

[4] T. Wang and D. Hobill, "Is the Preferred Basis selected by the environment?," arXiv:1412.2852 [quant-ph], 2014. [Online]. Available: https://arxiv.org/abs/1412.2852v1.

[5] B. Galvan, "On the preferred-basis problem and its possible solutions," arXiv:1008.3708 [quant-ph], 2010. [Online]. Available: https://arxiv.org/abs/1008.3708.

[6] H. Inamori, "No quantum process can explain the existence of the preferred basis: decoherence is not universal," arXiv:1609.07984 [physics.gen-ph], 2016. [Online]. Available: https://arxiv.org/abs/1609.07984.

[7] S. E. Misra B., "The Zeno's paradox in quantum theory," *Journal of Mathematical Physics*, vol. 18, no. 4, pp. 756-763, 1977.

[8] S. Viznyuk, "From QM to KM," 2020. [Online]. Available: https://www.academia.edu/41619476/From_QM_to_KM.

[9] J. Zhang and Y. Liu, "Fermi’s golden rule: its derivation and breakdown by an ideal model," arXiv:1604.06916 [quant-ph], 2016. [Online]. Available: https://arxiv.org/abs/1604.06916.

[10] S. Viznyuk, "Planck's law revisited," 2017. [Online]. Available: https://www.academia.edu/35548486/Plancks_law_revisited.

[11] S. Viznyuk, "Gravity-induced decay," 2016. [Online]. Available: https://www.academia.edu/21674738/Gravity-induced_decay.