The origin of unitary dynamics

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I argue the unitary evolution of quantum state is imposed by implicit extraction of classical information. I show the generating self-adjoint operator of time-driven unitary transformation is entropy, not Hamiltonian

The description of physical phenomena in terms of measurement outcomes, i.e., objective facts, vs. inferred possibilities, provides the principal distinction between the realm of classical physics, and the realm of quantum mechanics (QM) [1]. While former operates with definite values of observables, [which can only arise as a result of measurement], the latter derives the corresponding measurement probabilities, for a known device configuration (measurement eigenbasis)[1]. The concept of a *device* is pivotal for both realms. The device is both, the source of classical information contained in measurement events, and the reference frame (measurement eigenbasis), in which events are obtained.

Typically, there are two devices to consider: preparation device, and measuring device. The output of preparation device serves as input to measuring device. The output of a device is, by definition[2], one of its eigenstates. As eigenbasis of measuring device may not coincide with eigenbasis of preparation device, the projection $\boldsymbol{a}_{||}$ of preparation device' output $\boldsymbol{a}$ onto eigenspace of measuring device is a superposition[3] of measuring device' eigenvectors $\{\boldsymbol{b}\}$:

$$|\boldsymbol{a}\rangle = |\boldsymbol{a}_{\perp}\rangle + |\boldsymbol{a}_{\parallel}\rangle = |\boldsymbol{a}_{\perp}\rangle + \sum_{\boldsymbol{b}\in\boldsymbol{G}} |\boldsymbol{b}\rangle\langle\boldsymbol{b}|\boldsymbol{a}\rangle \tag{1}$$

, where $\boldsymbol{G}$ is a set of all possible mutually exclusive (orthogonal) measurement events.

When we express $\boldsymbol{a}$ as superposition (1) of eigenvectors $\{\boldsymbol{b}\}$ of measuring device with defined amplitudes $\langle\boldsymbol{b}|\boldsymbol{a}\rangle$, it means we already possess[4] some [classical] information about $\boldsymbol{a}$. Generally, however, such information can only be extracted by the measurement, as an event sample [2]. This information is only enough to determine event probabilities $|\langle\boldsymbol{b}|\boldsymbol{a}\rangle|^2$, in $N \to \infty$ limit, where N is the measurement sample size. It means, no measurement can completely determine input to measuring device [3, 4]. The description, based on extracted classical information, cannot be complete. It is the core problem which prompted development of QM during first half of 20th century. While QM was not able to resolve this (measurement) problem, prompting further incompleteness claims [5], it succeeded in predicting dependence of event probabilities $|\langle\boldsymbol{b}|\boldsymbol{a}\rangle|^2$, and associated expectation values, on parameter-driven change of measurement basis.

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[1] The adjective *known* always implies classical information. As pertaining to device configuration, the very notion of it being *known*, necessitates the classicality of [measuring] device [1]

[2] A device is completely defined by its eigenbasis, i.e., by a set of distinct states device can be in

[3] The *quantum state* thus refers to the description of preparation device' output in terms of measuring device' *eigenvectors* [14]. The phenomenon of quantum superposition [of measuring device' eigenvectors] arises from inability of measuring device to distinguish eigenvectors of preparation device

[4] The amplitudes $\langle\boldsymbol{b}|\boldsymbol{a}\rangle$ are known exactly if experimenter is in control of both, preparation, and measuring devices, meaning the experimenter determines the output of preparation device, and the measurement eigenbasis

If an event probability in old measurement basis $P(\boldsymbol{b}|\boldsymbol{a}) = |\langle \boldsymbol{b}|\boldsymbol{a}\rangle|^2$, in new $\boldsymbol{c} = \boldsymbol{V}\cdot\boldsymbol{b}$ basis $P(\boldsymbol{c}|\boldsymbol{a}) = |\langle \boldsymbol{c}|\boldsymbol{a}\rangle|^2 = \left|\langle \boldsymbol{b}|\boldsymbol{V}^{\dagger}\boldsymbol{a}\rangle\right|^2$. It shows that if transformation $\boldsymbol{V}$ is applied to measuring device, the expectation values are the same as if adjoint transformation $\boldsymbol{V}^{\dagger}$ is applied to preparation device, and v.v. A transformation[5] of quantum state is, therefore, indistinguishable from transformation of measurement basis.

The knowledge of basis transformation is crucial for QM analysis. Besides linearity, there is no restriction on transformation $\boldsymbol{V}$ considered above. The restriction appears once a *third party* (an *observer*) is introduced, with a premise that event probability $|\langle \boldsymbol{b}|\boldsymbol{a}\rangle|^2$ is invariant with respect to transformation $\boldsymbol{U}$ of observer basis relative to both, measuring, and preparation device:

$$|\langle \boldsymbol{b}'|\boldsymbol{a}'\rangle|^2 = |\langle \boldsymbol{U}\boldsymbol{b}|\boldsymbol{U}\boldsymbol{a}\rangle|^2 = \left|\langle \boldsymbol{b}|\boldsymbol{U}^{\dagger}\boldsymbol{U}|\boldsymbol{a}\rangle\right|^2 = |\langle \boldsymbol{b}|\boldsymbol{a}\rangle|^2 \quad \forall\, \boldsymbol{a}, \boldsymbol{b} \tag{2}$$

The independence of an event [probability] on observer [basis] is a defining property of *objective fact*. For *objectivity* to hold, the change $\boldsymbol{U}$ of observer basis has to be unitary or anti-unitary operation[6]. The *objectivity* imposes restriction on how device transformation $\boldsymbol{V}$, considered in previous paragraph, alters with a change of observation basis. Without objectivity restriction we have: $\boldsymbol{c}' = \boldsymbol{V}'\cdot\boldsymbol{b}' \;\Rightarrow\; \boldsymbol{U}\boldsymbol{c} = \boldsymbol{V}'\cdot\boldsymbol{U}\boldsymbol{b} \;\Rightarrow\; \boldsymbol{c} = \boldsymbol{U}^{-1}\boldsymbol{V}'\boldsymbol{U}\cdot\boldsymbol{b} \;\Rightarrow\; \boldsymbol{V} = \boldsymbol{U}^{-1}\boldsymbol{V}'\boldsymbol{U}$, i.e., $\boldsymbol{V}$ alters by similarity transformation. The *objectivity* restricts similarity to unitarity $\boldsymbol{V} = \boldsymbol{U}^{\dagger}\boldsymbol{V}'\boldsymbol{U}$, if the change of device' basis is relative to a third party (observer). The presence of third party is a crucial element of unitary dynamics which we shall investigate further below. The unitarity is imposed not by the mere presence of observer [6], but by the extraction of classical information, implied by the presence of observer. The classical information is carried by the event sample, in a form, e.g., of event probabilities (2).

The generator of a unitary transformation $\boldsymbol{U}$ is a self-adjoint operator $\boldsymbol{\mathcal{H}}$, i.e., $\boldsymbol{U} = exp(i\cdot\boldsymbol{\mathcal{H}})$. If basis transformation is parameter-driven, i.e., $\boldsymbol{\mathcal{H}} = \boldsymbol{\mathcal{H}}(t)$, then:

$$\boldsymbol{a}(t) = \boldsymbol{U}(t)\boldsymbol{a}(0) = exp\big(i\cdot\boldsymbol{\mathcal{H}}(t) - i\cdot\boldsymbol{\mathcal{H}}(0)\big)\boldsymbol{a}(0) \tag{3}$$

In differential form, (3) becomes:

$$i\frac{\partial \boldsymbol{a}}{\partial t} = \boldsymbol{H}\boldsymbol{a} \tag{4}$$

, where $\boldsymbol{H}(t) = -\,\partial\boldsymbol{\mathcal{H}}(t)/\partial t$. One would readily recognize (4) as Schrödinger equation [7], for an arbitrary parameter t. If t is a distance $t \equiv x$; and $\widehat{\boldsymbol{p}} \equiv \partial\boldsymbol{\mathcal{H}}(x)/\partial x$, then (4) becomes

$$i\frac{\partial \boldsymbol{a}}{\partial x} = -\widehat{\boldsymbol{p}}\boldsymbol{a} \tag{5}$$

The Schrödinger equation in a form (5) serves as definition of momentum operator $\widehat{\boldsymbol{p}}$. Any parameter-driven unitary transformation, where generator $\boldsymbol{\mathcal{H}}(t)$ is a differentiable function of the parameter, can be written in a form of Schrödinger equation. In case t is *time*, it was postulated [7] by Schrödinger, that $\boldsymbol{H}$ is *Hamiltonian*. As of now, it still remains one of QM postulates [4, 8].

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[5] The "*evolution of quantum state"* is a popular, albeit misleading, term

[6] From (2), $\boldsymbol{U}^{\dagger}\boldsymbol{U} = exp(i\varphi)\cdot\boldsymbol{I}$, where φ is real and $\boldsymbol{I}$ is identity operator. Since $\boldsymbol{U}^{\dagger}\boldsymbol{U}$ is self-adjoint, $exp(i\varphi) = \pm 1$

No theory is complete if it's based on postulates, i.e., if it cannot explain itself. Below, I show that if t is time, then $(\partial \boldsymbol{\mathcal{H}}(t)/\partial t = -\boldsymbol{H})$ is entropy operator.

Generally, measurements are done in preparation + measurement cycles (*PMC*). The *PMC*s produce statistical ensemble needed for determination of probabilities and expectation values.

Consider the preparation and measuring devices have the same eigenspace, i.e., $\boldsymbol{a}_\perp \equiv 0$ in (1). It means the total measurement probability is 1. The *PMC* output is one of eigenvectors $\{\boldsymbol{b}_i \in \boldsymbol{G}\}$, $1 \leq i \leq M$ of measuring device, where M is the dimension of measurement basis. The full output of measuring device is given by $\boldsymbol{b}_1^{\otimes n_1} \otimes \boldsymbol{b}_2^{\otimes n_2} \dots \otimes \boldsymbol{b}_M^{\otimes n_M}$ [3, 9], or, in Fock representation, by $|n_1, n_2, \dots, n_M\rangle$, where n_i is the number of occurrences of event $\boldsymbol{b}_i$; $N = \sum_{i=1}^{M} n_i$. The event sample $\{n_i\}$ contains classical information which has to persist in some encoded form. The encoded information constitutes the final result of the measurement, not the transient output sample[7]. The number of symbols needed to encode output information is given by Boltzmann's entropy H_Ω [2]:

$$H_\Omega(nats) \equiv \ln \Omega = \ln\left[\frac{N!}{\prod_i n_i!}\right] = \ln \Gamma(N+1) - \sum_i \ln \Gamma(n_i + 1) \qquad (6)$$

, where $\Omega = N!/(\prod_i n_i!)$ is the statistical weight of the sample[8]. As has been shown elsewhere [10], $H_\Omega < N \ln M$, i.e., for a finite-size sample, the number of symbols needed to encode measurement output is less than the size of the sample expressed in the same units. In units of $\log_\Omega$, the extracted information is represented by 1 symbol of cardinality Ω, i.e., $H_\Omega \equiv 1(\log_\Omega)$. Each of Ω possible values of this symbol, corresponds to a quantum state (1) with certain phase relationship between $\langle \boldsymbol{b}|\boldsymbol{a}\rangle$ amplitudes (1). Such $\boldsymbol{\Omega}$-states constitute eigenbasis in Fock space of event sample. The $\boldsymbol{\Omega}$-states are eigenvectors of entropy operator $\boldsymbol{H}_\Omega$, with the same eigenvalue $H_\Omega = 1(\log_\Omega) = \ln \Omega \ (nats) = \log_2 \Omega \ (bits)$. The entropy operator, in respective units, is then:

$$\boldsymbol{H}_\Omega = \boldsymbol{I}_\Omega \ (\log_\Omega) = \boldsymbol{I}_\Omega \cdot \ln \Omega \ \ (nats) = \sum_{i=1}^{\log_2 \Omega} (|0_i\rangle\langle 0_i| + |1_i\rangle\langle 1_i|) \ \ (bits) \qquad (7)$$

, where $\boldsymbol{I}_\Omega$ is the identity operator, acting in Fock space spanned by $\boldsymbol{\Omega}$-states. Different units of entropy (7) correspond to different encoding symbols, e.g., *bits* or *nats*. If encoded in *bits*, $\boldsymbol{\Omega}$-state is represented[9] by $\log_2 \Omega$ qubits $\{\boldsymbol{q}_i\}$:

$$|\boldsymbol{\Omega}\rangle = |\boldsymbol{q}_1 \boldsymbol{q}_2 \dots \boldsymbol{q}_{(\log_2 \Omega)}\rangle \qquad (8)$$

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[7] The eigenvectors $\{\boldsymbol{b}_i\}$ are not classical outcomes; rather, they can be thought of as radiation modes

[8] The Boltzmann's entropy is the correct measure in this case, as opposed to von Neumann's [15] entropy $S = -Tr(\rho \ln \rho)$ $(nats/event)$. Von Neumann's entropy cannot be defined from measurement events, because it requires knowledge of density matrix $\rho = |\boldsymbol{a}\rangle\langle\boldsymbol{a}|$, unattainable from the measurement [3]. Diagonal elements of ρ can be evaluated as $\rho_{ii} = (n_i/N)|_{N\to\infty}$. However, without off-diagonal elements, ρ would represent a mixture, not the pure quantum state (1). Therefore, von Neumann's entropy is not the eigenvalue of Fock state $|n_1, n_2, \dots, n_M\rangle$

[9] The encoded information can only be represented by integer number of symbols. The value $\log_2 \Omega$ can be non-integer, depending on cardinality of encoding alphabet, which introduces some noise into encoded information. Therefore, the encoded information may not unambiguously represent the event sample. It adds to be fact that event sample itself does not unambiguously represent the input to measuring device

, where qubits $\{\boldsymbol{q}_i\}$ are in definite $|\mathbf{0}_i\rangle, |\mathbf{1}_i\rangle$ *bit* states. From (7), $\boldsymbol{H}_\Omega|\mathbf{\Omega}\rangle = H_\Omega|\mathbf{\Omega}\rangle = |\mathbf{\Omega}\rangle \cdot \log_2 \Omega$.

The fact that $\mathbf{\Omega}$-states are bit product states, and not entangled qubit states, allows for derivation of parameter-driven unitary transformation between $\mathbf{\Omega}$-states. Different parameter values correspond to different observation bases. The parameter-driven transformation of observation basis is limited to Fock space of event sample by the objectivity condition (2).

The transformation of observation basis, considered so far, may seem a continuous function of the parameters, especially if written in a form of Schrödinger equation (4). However, the parameter values can only arise as a result of measurement, even if implied measurement. The continues values of the parameters would indicate a possibility of continuous measurement, with infinitely small intervals. That is an improbable proposition, from multiple perspectives, e.g., from uncertainty relations. The discreteness of $\mathbf{\Omega}$-states forces parameters to only take discrete values.

Any $2D$ self-adjoint operator is expressed [4] in terms of spacetime 4-vector $(t, \boldsymbol{r})$ as

$$\boldsymbol{\mathcal{H}}(t, \boldsymbol{r}) = t \cdot \boldsymbol{I} + (\boldsymbol{r}, \boldsymbol{\sigma}) = t \cdot \boldsymbol{I} + \theta \cdot (\boldsymbol{u}, \boldsymbol{\sigma}) \tag{9}$$

, where $\boldsymbol{r} = (x, y, z)$; $\boldsymbol{u} = \boldsymbol{r}/|\boldsymbol{r}|$; $\theta = |\boldsymbol{r}|$; $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are Pauli matrices. Operator (9) generates qubit's unitary transformation [4]:

$$\boldsymbol{U}(t, \boldsymbol{r}) = exp(i\boldsymbol{\mathcal{H}}) = exp(i \cdot t) \cdot (\boldsymbol{I} \cdot \cos\theta + i \cdot (\boldsymbol{u}, \boldsymbol{\sigma}) \cdot \sin\theta) \tag{10}$$

Any two qubits differ by unitary transformation (10), i.e., by real-valued classical parameters $(t, \boldsymbol{r})$: $(\forall \boldsymbol{q}_i)(\forall \boldsymbol{q}_j)\ (\exists\ t, \boldsymbol{r})(\boldsymbol{U}(t, \boldsymbol{r})\boldsymbol{q}_i = \boldsymbol{q}_j)$. The 4-vector $(t, \boldsymbol{r})$ is the relative spacetime position of the qubit. The qubits, making up $\mathbf{\Omega}$-states, give rise to spacetime as an entity. The change (10) of observation basis may only transform these qubits between $|\mathbf{0}\rangle, |\mathbf{1}\rangle$ bit states. It forces 4-vector $(t, \boldsymbol{r})$ to only take discrete values, i.e., *spacetime is quantized*.

Any two $\mathbf{\Omega}$-states differ by a tensor product of individual qubits' unitary transformations:

$$\boldsymbol{U}_\Omega = \boldsymbol{U}_1(t, \boldsymbol{r}_1) \otimes \boldsymbol{U}_2(t, \boldsymbol{r}_2) \ldots \otimes \boldsymbol{U}_{H_\Omega}(t, \boldsymbol{r}_{H_\Omega})\ ;\ H_\Omega = \log_2 \Omega \tag{11}$$

Here $\boldsymbol{U}_k(t, \boldsymbol{r}_k) = exp(i\boldsymbol{\mathcal{H}}_k(t, \boldsymbol{r}_k))$, where subscript k refers to parameters of, and to operators acting on, qubit $\boldsymbol{q}_k$; $\boldsymbol{r}_k$ are space coordinates of individual qubits. The unitary transformation (11) between given $\mathbf{\Omega}$-states is by no means unique. We are looking for the one which has time-driven part separated, as in (10). With (9), the tensor product (11) expands into

$$\boldsymbol{U}_\Omega = exp\left(i \sum_{k=1}^{H_\Omega} [t\boldsymbol{I}_k + (\boldsymbol{r}_k, \boldsymbol{\sigma}_k)]\right)\ ;\quad \boldsymbol{I}_k = |0_k\rangle\langle 0_k| + |1_k\rangle\langle 1_k|\ \ ;\ \ \boldsymbol{\sigma}_k = (\sigma_x, \sigma_y, \sigma_z)_k \tag{12}$$

Using entropy operator (7), I write (12) as

$$\boldsymbol{U}_\Omega(t, \boldsymbol{r}) = exp\left(i \cdot t \cdot \boldsymbol{H}_\Omega + i \cdot \sum_{k=1}^{H_\Omega} (\boldsymbol{r}_k, \boldsymbol{\sigma}_k)\right) \tag{13}$$

The above shows the generating self-adjoint operator for time-driven unitary transformation is entropy operator $\boldsymbol{H}_\Omega$. The underlying reason is: the Fock state $|n_1, n_2, \dots, n_M\rangle$ is the eigenstate of entropy, not of Hamiltonian. Unlike entropy (6), the [information] energy $\mathcal{E}\big((n_i);\ N, (p_i)\big)$ is not determined solely from event sample $\{n_i\}$. It requires additional information, in a form of event probabilities $\{p_i\}$, as evident from formula (6) in [2]:

$$\mathcal{E}\Big((n_i);\ N, \big(p_i\big)\Big) = \sum_{i=1}^{M}\left[\ln\frac{\Gamma(n_i+1)}{\Gamma\big(Np_i+1\big)} + \big(Np_i - n_i\big)\cdot\ln p_i\right] \tag{14}$$

In (14), the knowledge of probabilities $\{p_i\}$ is inseparable from the knowledge of quantum state proper. The concept of *knowledge* is based on entropy as measure of missing information. The entropy is the amount of *unknown*. The maximum entropy state has zero *known*. Thus, the amount of known, i.e., *knowledge*, equals difference between max entropy $H_\Omega^{max} = H_\Omega(\{n_i = N/M\}\ \forall\ i)$, and entropy (6) of event sample [11, 3, 2]:

$$\mathcal{L}_\Omega(\{n_i\}) = H_\Omega^{max} - H_\Omega(\{n_i\}) \tag{15}$$

If probabilities of events $\{\boldsymbol{b}_i\}$ are same[10], $p_i = 1/M\ \forall\ i$, the second term under sum in (14) disappears. The energy (14) then equals knowledge (15). The time-driven part of (13) becomes

$$\boldsymbol{U}_\Omega(t) = exp(i\cdot t\cdot \boldsymbol{H}_\Omega) = exp\big(i\cdot t\cdot(\boldsymbol{H}_\Omega^{max} - \boldsymbol{\mathcal{E}})\big) \tag{16}$$

Transformation (16) leads to traditional Schrödinger equation with Hamiltonian $\mathbb{H}_\Omega = \boldsymbol{\mathcal{E}} - H_\Omega^{max}$. The eigenvalues of $\mathbb{H}_\Omega$ are < 0, as expected, for bound states with discrete spectrum (14).

From (13), the spacetime quantization interval of $\boldsymbol{\Omega}$-states is by factor H_Ω smaller than quantization interval of standalone qubits. The use of differential Schrödinger equation (4), or any differentiation by classical parameters, may only be justified if H_Ω is sufficiently large, as in case of large event sample $\{n_i \gg 1\}$.

$\boldsymbol{\Omega}$-state represents classical information, obtained from the measurement, and encoded as spacetime configuration of qubits in definite $|\boldsymbol{0}\rangle, |\boldsymbol{1}\rangle$ bit states. $\boldsymbol{\Omega}$-states do not have phase. A phase would mean the presence of unencoded information. The phase is lost when measurement turns amplitude $\langle\boldsymbol{b}|\boldsymbol{a}\rangle$ into classical probability $|\langle\boldsymbol{b}|\boldsymbol{a}\rangle|^2$. However, the internal phase relationship between components of quantum state is not lost upon measurement. Consider a quantum state consisting of two components: $\boldsymbol{a} = \boldsymbol{a}_1 + \boldsymbol{a}_2$. The event $\boldsymbol{b}$ probability is[11]:

$$P(\boldsymbol{b}|\boldsymbol{a}) = |\langle\boldsymbol{b}|\boldsymbol{a}\rangle|^2 = P(\boldsymbol{b}|\boldsymbol{a}_1) + P(\boldsymbol{b}|\boldsymbol{a}_2) + 2\sqrt{P(\boldsymbol{b}|\boldsymbol{a}_1)P(\boldsymbol{b}|\boldsymbol{a}_2)}\cos\big(\varphi_{b,a_1} - \varphi_{b,a_2}\big) \tag{17}$$

It shows, the phase difference between two components of quantum state affects event probabilities, and, therefore, is converted into classical information by the measurement.

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[10] The equal probabilities $\{p_i\}$ are expected in canonical scenarios, as e.g., in case of double-slit experiment, where probabilities of a particle passing through either slit are equal; or, if, e.g., $\{p_i\}$ is a distribution of photons among radiation modes of the same energy

[11] Naturally, $P(\boldsymbol{b}|\boldsymbol{a}_1)$ and $P(\boldsymbol{b}|\boldsymbol{a}_2)$ would not be normalized to 1

I have shown the unitary dynamics is conditioned on extraction of classical information, implied by the presence of third party (observer). This is contrary to orthodox QM interpretations, which postulate unitary dynamics, and consequently, the Schrödinger equation, as an inherent property of quantum state. The conclusions of this paper, to a significant extent, are foretold by Bohr and Heisenberg [1, 6, 12, 13]. I have shown the generating self-adjoint operator of time-driven unitary transformation, in general, is entropy, not Hamiltonian. Perhaps the most intriguing finding is the emergence of spacetime as encoding structure for classical information.

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